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VON THÜNEN'S THEORY OF NATURAL WAGES

BY

H. L. MOORE

Johns Hopkins University, Baltimore, Md.

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VON THÜNEN'S THEORY OF NATURAL WAGES.

I.

THE CLASSICAL THEORY AND VON THÜNEN'S FORMULA.

No scientific work could be more beneficent than that which would solve the problem of natural or just wages. The clashing interests of the capitalist and laborer in the division of the product of industry result in conflicting claims in the distribution of that product, and the lack of a scientific solution of the problem renders it possible that each may claim a moral basis for his actions.

Several vigorous attempts have been made to throw light on this question; and, among those who have made important contributions to the subject, Thünen holds a conspicuous place. His theory contains much that is interesting and valuable; and the object of this paper is (1) to give this theory a critical consideration, and (2) to show his contribution to the theory of natural wages. * In order, however, that we may be placed in a position to form a correct estimate of Thünen's work, a brief review will be given of the theory of natural wages held by his English contemporaries.

The characteristic features of the classical theory of natural wages are exemplified in the opening paragraph of Ricardo's chapter on Wages. The first sentence of that paragraph shows the manner in which the classical economists reasoned upon the subject. "Labor," says Ricardo, "like all other things which are purchased and sold, and which may be increased or diminished in quantity, has its natural and its market price." Here we find it im-

* For the bibliography of Von Thünen's Theory, see last page.

plied that labor is a commodity; and it is stated that labor, like every other commodity, has its natural and its market price. The classical economists taught that the natural price of a commodity is its cost of production: hence they inferred that the natural price of labor is its cost of production. But, when they came to define the cost of production of labor, they met with difficulty, and, as Marx * has shown, substituted for the cost of production of labor the cost of production of the laborer.

The second sentence of Ricardo's paragraph presents two more features of the classical theory. "The natural price of labor," Ricardo states, "is that price which is necessary to enable the laborers, one with another, to subsist and to perpetuate their race, without either increase or diminution." Here we find, in the first place, that Ricardo made no attempt to consider the equity in the case. Although he admitted that the price of labor "varies at different times in the same country, and very materially differs in different countries," yet he made no attempt to discover the reasonable or just wages. He was concerned only with facts and with the operation of natural law. In the second place, we observe that he defined natural wages without reference to the product of the laborer. Natural wages, according to his definition, depend upon the requirements of the laborer. These, however, vary at different times and in different places with "the habits and customs of the people."

In summarizing, the following may be named as characteristic features of the classical theory of natural wages:

1. Labor was treated throughout as a mere commodity.

2. Natural wages were defined without reference to equity, the operation of natural law being the main fact considered.

3. Natural wages were defined without reference to the product of labor. The requirements of the laborer

*Marx, *Das Kapital*, Band I., 2 Auflage, pp. 599. *Cf.* Schmidt, *Der natürliche Arbeitslohn*, p. 13.

as limited by his surroundings were regarded as determining his natural earnings.

While these doctrines were being taught by the classical school, Thünen was working upon a theory that was in many respects new. He claimed that the teachings of economists concerning wages were based upon the existing system of distribution. But, because under the existing system the laborer might be had for the bare means of subsistence, he could see no reason on that account for calling the bare means of subsistence natural wages. He was profoundly convinced of the evils resulting to the laboring class in consequence of the prevailing theory; * and this fact was one of the chief reasons leading him to undertake to discover the just or equitable wages, the wages agreeable to the nature (*naturgemäss*) and to the destiny of man. †

To simplify his investigations, or, rather, to make his investigations possible, he makes use of the isolated state. This state, isolated from the rest of the world by means of a wilderness, is in a plain of uniform fertility. Its only city, in which are concentrated all of its non-agricultural industries, is located at its centre. It has neither railroads nor navigable waters, and perfect competition pervades the entire state. The study is based upon the supposition that the isolated state is in a static condition.

Before entering upon Thünen's mathematical work, we must get well in mind the meaning of the terms he uses.

1. He takes rye as his measure of value, and selects a Berlin *Scheffel* of that grain as his unit. 2. He considers all laborers of the same class equal in strength, skill, intel-

* The light in which he viewed the classical theory may be seen from one of his letters of 1830 in Schumacher's *J. H. v. Thünen: Ein Forscherleben*, p. 117.

† *Der isolirte Staat*, II. 1, p. 193: "Ja, ich habe gefunden das tiefere Eindringen in die Frage, Welches ist der naturgemässe Arbeitslohn? in den letzten Studien unmittelbar zu der Frage über die Bestimmung des Menschen führt."

ligence, etc. 3. In wages proper — *i.e.*, in the reward for labor itself (*für die Arbeit an sich*), and not including interest upon any capital that the laborer may possess in the way of household effects — he distinguishes two parts. One part is that required for the means of subsistence of the laborer and his family. This part is designated by the symbol a. The other part, designated by the symbol y, is the surplus that the laborer earns above his means of subsistence. The symbol $a + y$ is used to express the wages for one year's labor of a laboring family. 4. By "product of labor" he means that part of the gross product which is shared by the laborers and capitalists alone, that part of the gross product remaining after deduction of profits, costs of management, insurance, etc. This product, divided by the number of laborers employed, gives the "product of labor" of one man. He designates the "product of labor" of one man by the symbol p. 5. By capital he means a product of human labor, employed in production.

We are now ready to follow him in his work. He approaches the problem by asking whether higher wages can exist at the margin of cultivation in the isolated state than exist in real life. He supposes, for the sake of argument, that the wages of the laborers on the marginal farms are raised, and then tries to discover the result of the change.

The income from the marginal farms before wages were raised just covered wages and interest upon invested capital. No ground rent was paid. If now wages are raised, ground rent will become a negative quantity, and the cultivation of marginal lands can be continued only at a loss. Perfect competition, however, pervades the isolated state; and no producer will continue to cultivate marginal lands at a loss. Consequently, producers will no longer invest capital in making improvements; and, as soon as buildings and other improvements have become

dilapidated, they will abandon the marginal farms. The laborers from the marginal lands will then crowd into employments nearer the city. Competition sets in between the new laborers and the old; that is, between the laborers from the marginal farms and the laborers already employed nearer the city. But the old laborers already existed in such numbers that the product of the last laborer just covered the wages that he received. If the new laborers are to find employment, wages must be reduced below their former level. The attempt, therefore, to raise wages has worked to the laborer's detriment.

One might suppose, from this argument, that the laborer of the isolated state fares no better than the laborer of the modern state. But Thünen causes the reader to observe that he has reached the above conclusion only by supposing the rate of interest unalterable. If the rate of interest can be lowered, wages may be raised, and cultivation still continued upon marginal lands. The wages of the laborer in the isolated state are dependent upon causes determining the rate of interest. The knowledge, therefore, of the natural wages depends upon the knowledge of the law determining the connection between wages and interest.

I shall give here a brief outline of Thünen's course in the subsequent part of his work. His prime object is to get a mathematical expression for natural wages and for natural interest. To do this, he attempts first to get an algebraic equation expressing the interdependence of wages and interest. Having obtained this equation, he can proceed in any one of three ways to determine the unknown quantities; *i.e.*, the quantities representing wages and interest. He can find a second equation containing the unknown quantities, and then combine the two; or he can find an independent expression for interest, and then by substituting in the equation obtain wages; or else he can find an independent expression for wages, and then by making the substitution in the equation obtain inter-

est. We shall see that, having found the equation, he selects the last method just described to obtain the value of the two quantities.

But first the equation expressing the relation between wages and interest must be found. Thünen approaches this problem in § 13, a section that has been grossly misunderstood and misinterpreted. This section has a meaning only when considered in connection with the main object of Thünen's work. The problem that he has before him is to discover the natural wages of the *ordinary* laborer, a laborer without a store of capital to invest, having only his labor with which to secure his maintenance. In order to find the equitable or just wages of such a laborer, he places him under conditions where the rate of wages is dependent upon the laborer's own course of action. As, however, the laborer has no store of capital, the conditions under which he can secure natural wages must be conditions where, without a store of capital, he can determine the rate of wages. Now the purpose of § 13 is indicated in the heading of that section,— "The Reduction of the Efficiency of Capital to Terms of Labor" (*Reduktion der Wirksamkeit des Kapitals auf Arbeit*). Thünen tries to show how the co-operation of capital in the production of a commodity may be reduced to terms of labor, and in doing this he lays the groundwork for the development of the formula expressing the interdependence between wages and interest.

He proceeds as follows: If an amount of capital Q, expressed in terms of rye, dollars, or any other measure, is divided by the year's wages of a laborer $(a+y)$, the wages being expressed in the same terms as the capital, then we shall find "how large the capital is, expressed in years' labor * of a laboring family, or how many years' labor of

* There is an inaccuracy here that has confused Thünen's critics. By dividing capital by wages, we do not find the value of the capital expressed in terms of years' labor, but in terms of wages.

a laboring family a capitalist with Q capital can control." * An amount of capital equal in value to the year's wages of a laborer he considers a unit of capital: hence, if $\frac{Q}{a+y} = nq$, Q will represent nq units of capital. He supposes that the capitalist lends Q to an undertaker, who invests it in some industry. If he employs n laborers, each laborer will use $\frac{nq}{n} = q$ units of capital.

Now Thünen says, if from the gross returns of the industry "all expenses of the undertaker are deducted with the single exception of wages and interest, and if from the remainder business profit is subtracted, there is left the part of the product which we have called product of labor." † The "product of labor" of a man using q units of capital he designates by the symbol p. The question is, in what proportion will the capitalist and the laborer divide the "product of labor" between them? This question Thünen attempts to answer as follows: —

The n laborers employed in the industry bring forth a product np. Of this product the n laborers receive as wages $n(a+y)$. After the deduction of wages the capitalist receives as rent ‡ $n[p-(a+y)]$. The rent, divided by the capital employed, gives the rate of interest, which we shall designate by z.

$$\text{Then, } z = \frac{n[p-(a+y)]}{nq(a+y)} = \frac{p-(a+y)}{q(a+y)}.$$ §

Thünen changes the form of this equation somewhat in order to show the proportion in which the capitalist and the laborer share in the product p. From the equation $z = \frac{p-(a+y)}{q(a+y)}$ the following may be obtained: $qz(a+y) = p-(a+y)$; $(a+y)(1+qz) = p$; or,

$$1.\quad a+y = \frac{p}{1+qz} = \text{share of laborer.}$$

* p. 124. † p. 124.

‡ Thünen speaks of the earnings of capital as *Rente*, and the earnings of land as *Landrente*. These two terms I shall translate throughout this paper as rent and land rent, respectively.

§ p. 125.

To obtain the share of the capitalist, we must subtract from the product p the share of the laborer. Hence,

2. $p - \frac{p}{1+qz} = \frac{p+pqz-p}{1+qz} = \frac{pqz}{1+qz}$ = share of capitalist.

From equations 1 and 2, we can find the proportion in which the shares stand to each other: $\frac{p}{1+qz} : \frac{pqz}{1+qz} :: 1 : qz$. This result may be thus expressed: The earnings of one year's labor are to the earnings of q units of capital as 1 is to qz. Therefore, the earnings of one year's labor are to the earnings of one unit of capital as 1 is to z.

We shall have to follow Thünen but a little further to find how he applies the result which he has just obtained. He is going to make use of the law of substitution * as applied to labor and capital.

"In the production of one and the same product p, a part of the capital may be replaced by an increased amount of labor; and, *vice versa*, a part of the labor may be replaced by an increased amount of capital. Capital appears as a coworker, and enters into competition with the laborer. But it is in the power of the undertaker who employs n laborers with the capital Q, to give to the relative capital q, which one man uses, any desirable value † by increasing or diminishing n. The undertaker, knowing and following his interests, will increase the value of q just so much, until the costs of the work done by capital and the work done by men stand in direct proportion to their respective efficiency in production." ‡

Thünen therefore concludes that the respective efficiency of labor and capital is the measure of their earnings.

* Professor Marshall states that Thünen was the first to make use of this law in this connection. Cf. *Principles of Economics*, second edition, p. 556. Dr. Stuart Wood has recently applied this same law in the same relation with excellent effect. Cf. *Publications of American Economic Association*, vol. iv., No. 1.

† $\frac{Q}{\alpha+y} = nq$; the number of units of capital used by one laborer is $\frac{nq}{n}$; therefore, by increasing or diminishing n, q may be made to have any desirable value.

‡ p. 126.

But, as we have just seen, he has proved that the earnings of a unit of labor are to the earnings of a unit of capital as $1:z$. The important inference that he draws from these facts is thus stated: "The rate of interest z is the factor by means of which the relation of the efficiency of capital to that of human labor is expressed";* *i.e.*, the rate of interest z is the factor expressing the relative efficiency of capital and labor. Now observe his final conclusion: "We are herewith placed in a position to reduce to terms of labor the co-operation of capital in the production of a commodity."† In these words he states, as plainly as could be stated, the chief results of his investigations in § 13; and, although he claims that his advantage over Ricardo consists in his ability to reduce the co-operation of capital to terms of labor, the validity of that claim has not been tested.

A little reflection will show the fallacy that he has committed. His conclusion does not follow from his premises. In his premises he states: (1) that, "in the production of one and the same product p, a part of the capital may be replaced by an increased amount of labor," and "a part of the labor may be replaced by an increased amount of capital," and that "the undertaker, knowing and following his interests, will increase the value of q just so much, until the costs of the work done by capital and of the work done by men stand in direct proportion to their respective efficiency in production"; (2) that the earnings of a unit of labor are to the earnings of a unit of capital as $1:z$. He infers from these premises that the rate of interest z is the factor expressing the relative efficiency of capital and labor. But here he draws a general conclusion from particular premises. It

*p. 127.

†p. 127. Notice also this sentence, p. 127: "Durch diese Reduktion ist es dann möglich, die Produktionskosten eines Erzeugnisses, insofern keine Landrente darin enthalten ist, ganz in Arbeit auszudrücken, und die Arbeit wird dadurch wahrhaft zum Werthmesser für die Tauschgüter."

will be admitted that in the production of a given product *a part* of the capital may be replaced by labor, and *a part* of the labor replaced by capital. It will also be admitted that, in the *margin of indifference*, the wise undertaker will employ capital and labor in such proportions that their costs will be in proportion to their respective efficiency in production. But it cannot be admitted that, because in the *margin of indifference* the efficiency of capital may be reduced to terms of labor, therefore the entire efficiency of capital can be reduced to terms of labor, or that the efficiency of capital in the production of any commodity can be reduced to terms of labor, or that z expresses the relative efficiency of capital and labor. It expresses the relative efficiency only at the *margin of indifference*, the margin where capital and labor may be indifferently substituted for each other. If, then, z does not express the relative efficiency of capital and labor, Thünen cannot infer that, by means of z, the co-operation of capital in production can be reduced to terms of labor.

We shall see later what use Thünen makes of this supposed law, but now we shall return to the main problem under discussion. In the section that we have just considered he obtains the equation $a + y = \frac{p}{1+q^2}$. Although the quantities y and z occur in this equation, the expression in its present form is of no use; for the value of $(a + y)$ is dependent upon the value of z. Besides, p is not a constant quantity, but varies as q increases or diminishes; and the values of y and z depend upon the value of p. p, y, and z are therefore functions of q. The problem, then, is to find the values of p, y, and z for a given value of q.

In order to solve this problem, Thünen refers us to the isolated state. In the isolated state the laborers have it in their power either to work for undertakers or to lay out farms for themselves on the margin of cultivation. If the laborers are to be deterred from laying out new

farms, and persuaded to continue in the service of their employers, their wages plus the interest on the capital needed to lay out a farm must be equal to what the "product of labor" of the farm would be after its completion. These conditions may be expressed mathematically. If wages $= a + y$, the rate of interest $= z$, "product of labor" of one man $= p$, the number of units of capital used by one man cultivating the new farm $= q$, whose value is $q\ (a + y)$, since each unit of capital has the value of one year's wages, then, in order for the laborer to be deterred from laying out the farm, this equation must exist: $(a + y) + q\ (a + y)\ z = p$;* or $(a + y) = \frac{p}{1 + qz}$; or $z = \frac{p - (a + y)}{q\ (a + y)}$. Here a, p, and q are known quantities, and y and z are unknown.

We have now obtained an equation expressing the interdependence of wages and interest. The next problem is to find an independent expression for wages. To enable him to find this expression, Thünen supposes that a number of laborers combine to lay out a farm on the margin of cultivation of the isolated state. This new farm is to be of the same character as those already existing in the state. The combination of laborers divide themselves into two groups, which we shall call group A and group B. The laborers in group A remain in the service of undertakers, and by means of the surpluses of their wages furnish the means of subsistence to the laborers of group B, who lay out the farm. In order to avoid confusion, we shall call the laborers in groups A and B capital-producers.†

In the course of a year the farm is completed, and la-

* It must be remembered that Thünen considers the isolated state under static conditions, for otherwise equilibrium would not be maintained even if the relation existed expressed by the equation $(a + y) + q\ (a + y)\ z = p$. In a progressive state the laborers would secure a large advantage in laying out marginal farms, owing to the growth in value of those farms, due to general progress. *Cf.* Professor J. B. Clark, "De l'Influence de la Terre sur le Taux des Salaires," p. 256 of *Revue d'Économie Politique*, 1890.

† Thünen calls the laborers in groups A and B *kapitalerzeugende Arbeiter* (p. 151).

borers are employed to cultivate it. The wages of each of these laborers must be so great that the income which each of them receives from his surplus when that surplus is placed at interest — *i.e.*, yz — will be equal to the sum received by each of the capital-producers as his share of the rent* of the farm; for, if this were not the case, the laborers would lay out farms for themselves.

These facts Thünen expresses in mathematical language, and is thereby enabled to obtain the formula: Natural wages $= \sqrt{ap}$. He uses the following symbols: —

Let $a + y =$ the year's wages of a laboring family, a and y retaining the same meanings that they have had throughout this paper.

$nq =$ the number of capital-producers in group B; that is, the number of men needed to lay out the farm. Thünen admits that, to lay out the farm, is needed not only labor, but also capital; but he summarily dismisses this difficulty by saying, "According to § 13, we can reduce the co-operation of capital to terms of labor" (p. 152). Later on I shall show that in the ascertainment of the quantity nq lies the error which vitiates the formula $\sqrt{ap}$.

As the farm is completed in one year, its value is equal to nq units of capital.

$anq =$ the amount consumed by the nq men in group B.

$\frac{anq}{y} =$ the number of capital-producers in group A, since the capital-producers in group A support those in group B by means of their surpluses.

Then $nq + \frac{anq}{y} = \frac{nq\,(a+y)}{y} =$ total number of capital-producers in groups A and B.

$n =$ number of men employed to cultivate the farm after its completion.

$n\,(a + y) =$ total wages paid n men.

$p =$ "product of labor" of one man, working with q units of capital.

$np =$ total "product of labor."

$np - n\ \ (a + y) = n\ \ [p - (a + y)] =$ total income from the farm for one year, or, as Thünen calls it, the rent of the farm. This rent is the property of $\frac{nq\,(a+y)}{y}$ men.

* Rent is used here in the sense noted above, and means earnings of capital, not land rent.

Therefore $\frac{n[p-(a+y)]}{\frac{nq(a+y)}{y}} = \frac{[p-(a+y)]y}{q(a+y)}$ = share of each of the capital-producers, or his portion of the rent of the farm. In this last expression all the quantities are known excepting y.

Now, it has already been observed that, for the laborers to be induced to work on the new farm, the income which each receives from his surplus when that surplus is placed at interest must be equal to the rent received by each capital-producer: hence yz must be equal to $\frac{[p-(a+y)]y}{q(a+y)}$. Capital-producers and laborers, therefore, have a common interest in making the above function as large as possible; and, when the function has reached its maximum value, the interests of capital-producers and laborers will be satisfied. The question is, then, For what value of y will $\frac{[p-(a+y)]y}{q(a+y)}$ have a maximum value?

Differential calculus enables Thünen to answer this question. He solves the problem by differentiating the function with respect to y, and then placing the differential equal to zero. He finds that the function will have a maximum value when $(a+y) = \sqrt{ap}$.

Having obtained the formula for natural wages,* he is able to find the expression for the natural rate of interest by substituting $\sqrt{ap}$ for $(a+y)$ in the equation $z = \frac{p-(a+y)}{q(a+y)}$.

* Thünen placed such great value upon the formula $\sqrt{ap}$ that he requested it should adorn his tombstone. According to Schumacher (*J. H. v. Thünen: Ein Forscherleben*, p. 322) his wish was respected: "Die Krone seiner Gesetze, das Resultat mühseliger Untersuchungen über das Verhältniss des Arbeitslohns zum Zinsfuss und zur Landrente, wie solches aus seinem Arsenale mathematischer Formeln siegreich hervorging:

$$\text{Der naturgemässe Arbeitslohn} = \sqrt{ap}$$

schmückt als Denkspruch seinen einfachen Grabstein im Hügellande von Mecklenburg, wie er in schöner Stunde selber gewünscht."

II.

CRITICISMS OF THE FORMULA: NATURAL WAGES $= \sqrt{ap}$.

THÜNEN'S theory of wages holds a peculiar place in German Economics. Important as the theory is, it is unmentioned in the majority of German works,—a fact which Falck and Komorzynski attribute to the abstruse mathematical character of his method. But even among the few critics that Thünen has had there is a lack of harmony concerning the correctness of his work: some accept his mathematical results in their entirety; others accept a part, and refuse the rest; while still others reject his results *in toto*.

In the following pages my purpose is to endeavor to show that those critics who have offered the strongest objections to the correctness of Thünen's results have either overlooked the limited premises from which he started, or forgotten the hypothetical nature of his conclusions. At the end I shall try to expose the error which, so far as Thünen's own work is concerned, vitiates the correctness of his formula.

1. Let us examine first the position of Falck, who was one of the last to devote a monograph to Thünen's work. The main object of Falck's dissertation is to prove the mathematical inaccuracy in the formula for natural wages. He claims that the formula $\sqrt{ap}$ "is the keystone of Thünen's whole system"; and, if you reject it, his "system loses all practical importance." *

* Falck, *Die Thünen'sche Lehre vom Bildungsgesetz des Zinsfusses und vom naturgemässen Arbeitslohn*, p. 32.

He proceeds in his argument as follows: Thünen obtained the formula $\sqrt{ap}$ from the expression $\frac{[p-(a+y)]\,y}{q\,(a+y)}$. If we represent this expression by R, then

$$R = \frac{[p-(a+y)]\,y}{q\,(a+y)} = \frac{py}{q\,(a+y)} - \frac{y}{q} = \left[\frac{p}{a+y} - 1\right]\frac{y}{q}.$$

From the last expression we find, when $y = o$, the value of R becomes o. In other words, when wages are reduced to the bare means of subsistence, the rent of the capital-producer vanishes. This fact seems to startle Falck; and, forgetting that Thünen's work has nothing to do with actual conditions, but is based upon purely hypothetical conditions, he claims that the phenomenon indicated by the expression can have no economic cause, "since we can daily convince ourselves that, as a matter of fact, in many places wages are reduced not only to the bare means of subsistence, but even below that amount, while the income of the capitalist not only does not vanish, but may increase." * He infers, therefore, that there is a mathematical error in the expression; and he claims to prove the error by the following argument: —

"The formula $\frac{[p-(a+y)]\,y}{q\,(a+y)}$ was obtained from the formula $\frac{\frac{n\,[p-(a+y)]}{nq\,(a+y)}}{y}$. The numerator denotes the rent from the farm, the denominator the number of those among whom the rent is divided. But is the y of the denominator really equivalent to the y of the numerator?" He denies that the y's are equivalent, asserting that the "y of the numerator denotes the surplus that is paid to the laborer at this particular time; but the y of the denominator denotes the surplus of wages that existed before the laying out of the farm." "Only by placing these two y's equal to each other," he claims, "has it been possible for the rent to obtain a maximum value at a definite rate of wages." †

Falck is under the impression that the rate of wages which exists before the laying out of the farm is different

* Falck, p. 34. † *Ibid.*, p. 35.

from the rate that exists after the farm is completed. As this is an error into which several critics have fallen, it will be well to show that, in supposing a number of laborers to combine in laying out a marginal farm, Thünen proceeds on the assumption that natural wages exist in the isolated state before the farm is begun, and that the supposition is introduced merely as a means of ascertaining the mathematical expression for the prevailing rate of wages. Thünen attempts to place such conditions upon the isolated state, that natural wages not only can be realized, but are realized; and his main problem is to find the mathematical expression for such wages.

Let us briefly review the chief limitations that he places upon the isolated state. He assumes that the isolated state is in a static condition; that the laborers are equal in intelligence, skill, etc.; and that perfect competition pervades the entire state. He assumes that beyond the margin of cultivation there is a limitless territory whose fertility is equal to that of lands already under cultivation, and he maintains that the rate of wages and the rate of interest existing at the margin of cultivation determine the rate of wages and rate of interest throughout the entire state. With these conditions placed upon the isolated state, Thünen claims that the mere possibility * of laborers laying out farms for themselves will compel the undertakers to pay laborers wages that will be equal to what the latter could earn by laying out farms and cultivating them on their own responsibility.

He argues (pp. 146, 147), if the undertakers should attempt to lower wages, laborers would emigrate to the margin, and begin cultivation on their own responsibility. Since, however, the number of laborers is constant, this act on their part would cause scarcity of labor in the interior of the state, which would result in a loss to the

* "Die blosse Möglichkeit für die Arbeiter, sich in der Wildniss anzusiedeln ohne dass dies That wird," etc. *Der isolirte Staat*, Part II., p. 147.

undertakers. If, therefore, the undertakers desire to keep their laborers, they must pay them such wages as would make emigration to the margin of the state unprofitable.

Falck might claim that Thünen's supposition of a party of laborers combining to lay out a farm is an illustration of the manner in which the laborers would proceed to enforce higher wages when undertakers had reduced them below the natural limit. But this claim could not be sustained; for Thünen distinctly tells us that his investigations rest upon the supposition that the isolated state is in a static condition (*im beharrenden Zustand*, p. 146). By static condition he does not mean that the laborers are at war with their employers, trying to obtain natural wages. He means that they already receive natural wages. This follows from what he himself says concerning the static condition: "Im isolirten Staat haben wir . . . stets den endlichen Erfolg, also das erreichte Ziel, vor Augen gehabt. Mit dem erreichten Ziel tritt Ruhe und damit der beharrende Zustand ein; und hier erblicken wir Gesetzmässigkeit, während in der Uebergangsperiode Manches uns als ein unentwirrbares Chaos erscheint" (p. 35). He believes that natural wages already exist in the isolated state; and, as a means of discovering the mathematical expression for such wages, he supposes that a number of laborers, to whom it is a matter of indifference whether they labor for wages or cultivate a marginal farm on their own account, combine to lay out a farm.

We can now easily see the fallacy in Falck's objection. Falck rests his whole proof of the inaccuracy of Thünen's formula on his claim that the y of the denominator of the expression $\frac{n[p-(a+y)]}{\frac{nq(a+y)}{y}}$ has a different value from the y of the numerator. The sole reason that he offers to substantiate his claim is that the y of the denominator represents

the surplus that the laborer receives before the farm is begun, and the y of the numerator represents the surplus that he receives after the farm is completed. But this is no reason whatever why the y's are not equal. Because y represents two quantities, it by no means follows that those quantities are unequal. If the analysis that I have given of Thünen's method of procedure is correct, natural wages are assumed to exist in the isolated state both before and after the completion of the marginal farm; and consequently the y's in the above expression are equal.

2. Roscher* claims that even in the isolated state, Thünen's formula $\sqrt{ap}$ does not represent natural wages; for, he says, if labor and capital are combined in different proportions in different industries, the laborers will not be justly rewarded if all receive wages $= \sqrt{ap}$. For example, if an artist, using cheap fuel, makes valuable vases out of cheap clay, it is not just or natural that he should receive wages equal to or less than the wages of an ordinary laborer. But Professor Roscher claims that the formula $\sqrt{ap}$ would bring about this relation, because a has the same value for the artist and the laborer, and p varies with the amount of capital used. Hence, when the laborer is employed in an industry where much capital is used, "product of labor," p, will be very great; and, if his wages $= \sqrt{ap}$, he may receive more than the artist who uses only a small amount of capital.

Komorzynski† holds that Roscher is right in saying that $\sqrt{ap}$ does not represent natural wages, where labor and capital combine in different proportions in production; but he shows that Roscher's illustration is defective, since he compares laborers of different classes, an artist and an ordinary laborer, whereas Thünen's investigations are concerned only with the ordinary laborer.

* Roscher, *Geschichte der Nationalökonomik in Deutschland*, p. 896.

† Komorzynski, "Thünen's naturgemässer Arbeitslohn," *Zeitschrift für Volkswirthschaft, Socialpolitik, und Verwaltung*, Dritter Band, 1 Heft, p. 53.

Now let us examine the argument offered by Roscher and approved by Komorzynski, that, where capital and labor combine in different proportions in production, $\sqrt{ap}$ does not represent natural wages even in the isolated state. In order to present the matter clearly, I shall use an illustration found in Thünen's own work,* where capital and labor combine in different proportions in production. In § 13 he announces the law that the price of commodities tends to conform to their cost of production, and attempts to illustrate the law by a comparison of the prices of mining and agricultural products. In the early part of his work he assumed that the mines of the isolated state lay in the neighborhood of the city; but, in order that he may illustrate his law by a comparison of the prices of silver and grain, he supposes, for the time being, that the silver mines are scattered about the state, that the last mine which is worked lies at the margin of the state, and that, further in the wilderness, mines of equal fertility to the marginal mine are found, but that they are not worked, because the product would not pay for the cost of production. This follows from the fact that the product of the marginal mine just covers the cost of production.

Now, it has already been observed that the product of the marginal farm just covers the cost of production. Then, since perfect competition pervades the isolated state, and laborers may work either at farming or mining, it follows that the wages of the farm laborer and the wages of the miner must have equal values. Hence, if we can find the wages of the former in terms of grain and the wages of the latter in terms of silver, we shall be able to find the exchange values of grain and silver at the margin of the isolated state.

Thünen's formula for wages is $a + y = \frac{p}{1+qz}$. He supposes that the rate of interest — which, of course, is the same for owners of mines and owners of farms — is five

* Part II., pp. 131, 132.

per cent. In the above formula, and, indeed, everywhere else in Thünen's work, p represents "product of labor" *in kind*. When the laborers are employed in silver mining, p represents a certain amount of silver. When they are employed in agriculture, it represents a certain amount of grain. Thünen supposes that in the case of the marginal silver mine p equals $7\frac{1}{2}$ pounds of silver, and in the case of the marginal farm p equals 240 scheffels of rye. He then says, "Since different industries require different amounts of capital, q will represent different quantities." * He supposes that in mining $q = 20$, and in agriculture $q = 12$. By making these substitutions in the formula, we find that the wages of the miner $(a + y) = \frac{7\frac{1}{2}}{1 + 20 \times \frac{1}{250}} = 3\frac{3}{4}$ pounds of silver; and the wages, of the farm laborer $(a + y) = \frac{240}{1 + 12 \times \frac{1}{250}} = 150$ scheffels of rye. $3\frac{3}{4}$ pounds of silver, therefore, has the same value as 150 scheffels of rye; and $7\frac{1}{2}$ pounds of silver, the "product of labor," p, in silver mining, has the same value as 300 scheffels of rye.

Now, Roscher and Komorzynski would say $\sqrt{ap}$ does not represent natural or just wages, because the miner would receive as wages $\sqrt{ap} = \sqrt{a \times 300}$; while the farm laborer would receive $\sqrt{ap} = \sqrt{a \times 240}$. Whatever the value of a may be, it is the same for both; and consequently the miner would receive more than the farm laborer. Upon the face of it, this argument looks sound; but yet we should scarcely expect Thünen to make this blunder after telling us that throughout the isolated state all laborers receive the same wages, and that in different industries the quantity q, the amount of capital used, is different. As a matter of fact, I think it may be proved that the error lies with Roscher and Komorzynski. They attach

* Part II., p. 131. Falck attempts to defend Thünen against the criticism of Roscher, and says: "Der Herr Verfasser [Roscher] scheint hier vergessen zu haben, dass die Arbeiter des isolirten Staats Alle mit dem gleichen Capital ausgerüstet sind und dass daher ein verschiedenes Quotverhältniss geradezu undenkbar ist" (p. 55). The quotation above from Thünen, together with his illustration of different amounts of capital used in silver mining and in agriculture, show the absurd position that Falck has taken.

an entirely wrong meaning to p when p occurs in the formula $\sqrt{ap}$. In the formula $\sqrt{ap}$, p means "product of labor" in agriculture at the margin of the isolated state, and in the formula $\sqrt{ap}$ it never means anything else.

In the beginning of his work Thünen gives a general definition of p as the "product of labor" of one man; that is, the product to be shared between the laborer and the owner of the capital which he uses. He proceeds with his work, and bases his investigations at times upon actual conditions, and at times upon the assumed conditions of the isolated state. In his investigations based upon actual conditions he develops the formula $z = \frac{p-(a+y)}{q(a+y)}$; but, as his object is to find the expression for the natural wages of the isolated state, he tells us that in its present form the expression $z = \frac{p-(a+y)}{q(a+y)}$ is of no use to him. But why? Because p does not represent a constant quantity, "but rises and falls with q. y and z, in turn, depend upon p. Hence p, y, and z are functions of q." * He then definitely and distinctly states the problem before him. "The problem is then," he says, "to find the value of p, y, and z for a given value of q." †

After announcing his problem, he immediately leaves his investigations based upon actual conditions, and goes to the isolated state. He then shows that, if the laborers are to be deterred from laying out marginal farms, this equation must exist $(a+y)+q(a+y)z=p$, where p equals the "product of labor" in agriculture on the marginal lands. "Here," he says, "a, p, and q are determinate, y and z indeterminate" (p. 141). With these quantities assumed as known, he proceeds with his investigations, and ends by declaring that natural wages $=\sqrt{ap}$. But what does p mean here? Evidently, it has but one meaning, the "product of labor" in agriculture at the margin of the isolated state.‡

* Part II., p. 139. † *Ibid.*

‡ Misconception with regard to the meaning of p as that quantity occurs in the formula $\sqrt{ap}$ has been a source of numerous errors, and it is important that

3. Komorzynski takes a peculiar position in his criticism of Thünen. Thünen tells us he considers his isolating method the most valuable part of his work;* but Komorzynski insists that the isolated state, with all its appurtenances, is an unnecessary part of Thünen's work, and serves merely to obscure his investigations and confuse his critics. Thünen believes that the formula for natural wages can be found only in one way; namely, by means of the conditions placed upon the isolated state.† But Komorzynski claims that Thünen unwittingly develops the formula in two ways: that the second method of obtaining the formula has nothing to do with the isolated state, but is based upon actual conditions; and, when the two methods are divested of all unnecessary suppositions, the

the point made above should be understood. Thünen's purpose is merely to investigate, under the favorable conditions of the isolated state, the influence of free land upon wages. His position is that, at the margin of cultivation in the isolated state, the determination of the rate of wages and the rate of interest will be in accord with the best interests of the laborer, and that the rate of wages and the rate of interest at the margin determine the rates throughout the state. If Thünen attempted to show anything, it was that wages at the margin equal $\sqrt{ap}$, where p equals "product of labor" in agriculture; and his claim is that the same wages will exist throughout the isolated state. "Wir behaupten, dass der an der Grenze des isolirten Staats sich bildende Arbeitslohn und Zinsfuss normirend für den ganzen Staat ist" (p. 142).

Thünen's assumption that the mines of the isolated state are situated near the city (Part I., p. 1) confirms the idea that it was his purpose to investigate under favorable conditions the influence of free land upon wages; for, if the mines had been so placed that the one whose product just covered its cost of production was situated at the margin of the state, and that a number of equally fertile mines were left untouched in the wilderness, then, instead of the laborers combining to lay out a farm on the margin, they would unite to work a mine. We might then make suppositions concerning the latter combination similar to those that Thünen made concerning the former combination; and, if we should use capital letters where Thünen used small ones, we should find, by following Thünen's method, that the wages of each miner would be $\sqrt{AP}$. But $A = a$, and P is greater in value than p, because more capital is used in the production of P than is used in the production of p. Hence $\sqrt{AP}$ is greater than $\sqrt{ap}$; and, consequently, it would be more profitable for the laborers to engage in mining than in agriculture. Thünen saw what the difficulty would be if he supposed the mines to be placed in this way; and for this reason he was careful, in making the illustration I have given in the text, to state that his supposition concerning the position of the mines was only temporary.

* Part I., p. xix. † Part II., p. 26.

ascertainment of the formula is found to rest upon two conditions which are common to the two methods.

The second method of obtaining the formula $\sqrt{ap}$ is, according to Komorzynski, as follows: In the part of his work based upon actual conditions Thünen develops the formula $z = \frac{p-(a+y)}{q(a+y)}$, and states that it expresses the rate of interest. In another part of his work he states that it is to the advantage of the laborer to have wages and interest bear such relation to each other that the laborer will receive the maximum income from his surplus when that surplus is placed at interest. Now, Komorzynski argues, since the surplus is y, and the rate of interest $z = \frac{p-(a+y)}{q(a+y)}$, the interests of the laborer will be subserved when yz, which is equal to $\frac{y\,[p-(a+y)]}{q(a+y)}$, obtains a maximum value. This takes place when $(a+y) = \sqrt{ap}$. He therefore concludes that Thünen's suppositions concerning the isolated state are all unnecessary, and all that is needed to prove that $\sqrt{ap}$ represents the best rate of wages for the laborer is to prove: —

A. That $z = \frac{p-(a+y)}{q(a+y)}$ expresses a general formula for interest.

B. That it is to the advantage of the laborer to have wages and interest bear such relation to each other that the laborer will receive a maximum income from his surplus when that surplus is placed at interest.

Why Thünen did not proceed in the manner indicated I shall consider later on. At present we shall consider Komorzynski's attempt to prove an error in the development of the formula $\sqrt{ap}$ on the ground that the conditions A and B are unsustainable.

A. He claims that the subtle error which vitiates the formula for natural wages is that $z = \frac{p-(a+y)}{q(a+y)}$ is not a general formula for the rate of interest. To fulfil the conditions of a general formula for the rate of interest, it must express the same rate of interest for all industries.

But Komorzynski says the above formula is not capable of doing this; for "wages $(a+y)$ and interest z are by means of Thünen's formula $z=\frac{p-(a+y)}{q(a+y)}$ brought into a relation which, in turn, varies with the changing quantities p and q. The quantity p, the surplus value of the product over the value of the capital consumed in production, and likewise the quantity q, . . . are different in different forms of production. From this it follows that the relation which, according to the formula, exists between the rate of interest z and the rate of wages $(a+y)$, will be different in different forms of production." * In other words, since the quantities p and q are different in different forms of production, he infers that the relation of $(a+y)$ and z expressed by $z=\frac{p-(a+y)}{q(a+y)}$ will be different in different forms of production; and therefore, since the formula does not express the same rate of interest for all industries, it is not a general formula for the rate of interest.

The error in the argument can best be exposed by means of the example that Komorzynski gives to illustrate his argument. The illustration is intended to show why the formula does not represent the general rate of interest. He says: "If the rate of interest is 5 per cent. and the rate of wages 400 florins, then, in three different forms of production, these equations may exist: —

I. $\frac{5}{100}=\frac{2{,}500-400}{105\times 400}$, where $p=2{,}500$ and $q=105$;
II. $\frac{5}{100}=\frac{1{,}200-400}{40\times 400}$, where $p=1{,}200$ and $q=40$;
III. $\frac{5}{100}=\frac{460-400}{3\times 400}$, where $p=460$ and $q=3$;

But, if $(a+y)$ should rise from 400 to 450, then the following unequal rates of interest would result: —

I. $\frac{4.33}{100}=\frac{2{,}500-450}{105\times 450}$; II. $\frac{4.16}{100}=\frac{1{,}200-450}{40\times 450}$; III. $\frac{.741}{100}=\frac{460-450}{3\times 450}$." †

* Komorzynski, p. 58.

† *Ibid.*, p. 59, note. There is a slight error in the original which I have corrected in the quotation given above. Where $\frac{.741}{100}$ occurs above, $\frac{7.41}{100}$ is found in the original.

This example, he claims, illustrates that the formula is not a general formula for the rate of interest, or, better, a formula for the general rate of interest, but that it represents the relation between wages and interest in specific industries.

The fallacy in the argument may be readily shown. Komorzynski has overlooked Thünen's definition of p. In his illustration Komorzynski makes p represent, in each of the three cases, a definite number of florins. This is evident from the fact that in each case he subtracts wages, expressed in terms or florins, from p. Furthermore, when he supposes wages to rise from 400 to 450 florins, he assumes that in each of the three cases p retains its value in terms of florins. In his illustration and throughout his whole article* he assumes that p represents a definite *value*. But Thünen distinctly defines p as "product of labor"; that is, "product of labor" in kind (pp. 80, 167). When laborers are employed in silver mining, p is expressed in terms of silver (p. 131); and, when they are employed in agriculture, p is expressed in terms of grain (p. 131). If, then, p means "product of labor" in kind, Komorzynski cannot assume that the value of p, or the price of p, expressed in florins, remains constant when the rate of wages changes. This error renders his objections useless; for if, after the rise of wages, the demands of the community require the continuance of production in groups I., II., III., the formula $z = \frac{p-(a+y)}{q(a+y)}$ (provided there is no other objection to the formula than that which Komorzynski offers) may still represent equal rates of interest for the three groups if p, remaining constant in quantity as "product of labor" in kind, may change its value or change its price in terms of florins.

B. Komorzynski also attempts to prove defective Thünen's supposition that it is to the advantage of the laborer

*For example, p. 58, "Die Grösse p, der Wertüberschuss des Productes," etc.; p. 55, "Hier bedeutet p den Tauschwert (erlangbaren Verkaufspreis) des Productes," etc.

to have wages and interest bear such a relation to each other that the laborer will receive a maximum income from his surplus when that surplus is placed at interest.* In brief, he claims that it is not to the interest of the laborer that *yz* should attain a maximum value.

His objection may be thus stated: The interests of laborers as to the relation of wages and interest vary according as they have saved during many or during few years. If they have saved during many years, they desire to have wages low and interest high. If, on the other hand, they are just beginning to save, they desire a higher rate of wages and a lower rate of interest. Hence there is no definite relation of wages and interest that, under all circumstances, is the best relation for the laborers. Concerning Thünen's supposition that it is to the advantage of the laborer to have *yz* attain a maximum value, Komorzynski says that it is based on the arbitrary assumption that the laborers save only during one year, and that all laborers have only an amount of capital equal to *y*.

This objection has much in its favor; but, before considering it, we must put Thünen in the right light. In introducing Komorzynski's criticism, it was stated that his position is peculiar, since he holds that, while Thünen believed he could obtain his formula for natural wages only in one way,— by means of the conditions of the isolated state,— he unwittingly develops it in two ways; that, when the two methods are disembarrassed of all superfluous suppositions, the ascertainment of the formula is found to rest upon two conditions, which are common to the two methods; and that the ascertainment of the formula by the second method is based, not upon the hypothetical conditions of the isolated state, but upon actual conditions. By the logic of his position, therefore,

* This question is also discussed by Knapp, G. F., *Zur Prüfung der Untersuchungen Thünen's über Lohn und Zinsfuss im isolirten Staate*, pp. 18–26; Roscher, *Geschichte*, etc., p. 896; Schmidt, C., *Der natürliche Arbeitslohn*, pp. 34–37.

he is forced to charge Thünen with holding that it is to the advantage of the laborer, under actual conditions, to have wages and interest bear such relation to each other that he will receive the maximum income from his surplus when that surplus is placed at interest. This charge cannot be proved. Thünen's hypothesis that there is a definite relation of wages and interest which is most desirable for the laborer is based upon the supposition that there is a direct interdependence between wages and interest. But he cannot be charged with claiming a direct interdependence between wages and interest under actual conditions,* for nowhere in his investigations based upon actual conditions does he make such a supposition. So far, however, as concerns the isolated state,—a state in a static condition, with constant capital, with constant population having constant wants and constant methods of production,—his claim of an interdependence of wages and interest cannot be denied.

Now, when the above objection is considered as directed towards Thünen's method of developing the formula on the basis of the isolated state, it has the merit, not of proving his investigations false, but of showing them to be painfully contracted and incomplete. In a state where there is a direct interdependence of wages and interest, it is true that, when laborers possess different amounts of capital, those who possess much will desire a different relation of wages and interest from those who possess little. It is also true that, when laborers have saved during many years, they desire a different relation of wages and interest from what they desired when they were just beginning to save. But, after we have admitted

*Komorzynski's error grows out of his misunderstanding of the formula $z=\frac{p-(a+y)}{q(a+y}$, as that formula occurs in § 13. *If* Thünen claimed that the formula represents a general formula for interest under actual circumstances, and *if* he intended p to represent a constant price, as Komorzynski understands it, then we might charge him with holding that there is a direct interdependence of wages and interest under actual circumstances.

all this, we cannot deny that the laborers who are just beginning to save will be interested in having the relation of wages and interest such that they will obtain a maximum income from their surplus when that surplus is placed at interest. If, then, y equals the surplus, and z the rate of interest, we cannot deny that, when laborers begin to save, it will be to their advantage to have yz obtain a maximum value. Komorzynski's argument does not prove an error in Thünen's work: it merely shows its incompleteness.

4. I shall now attempt to point out a fallacy in Thünen's reasoning that vitiates his formula; and, in order to expose this fallacy, I must briefly review the chief points in § 13, that repeatedly misinterpreted section of Thünen's work. As I have shown in the early part of this article, Thünen's prime object in § 13 is to find means by which he can reduce to terms of labor the co-operation of capital in production. He proceeds to do this by saying, if an amount of capital Q is divided by the year's wages of a laborer $(a+y)$, we shall find "how large the capital is, expressed in years' labor of a laboring family" ("Wie gross das Kapital in Jahresarbeiten einer Arbeiterfamilie ausgedrückt ist," p. 124). Although we see what Thünen means to say, yet his words do not express his meaning. By dividing capital by wages, he does not obtain an expression for capital in terms of years' labor of a laboring family, but in terms of wages. Further on in the same section (p. 128) he repeats the same idea by saying, in effect, that capital may be reduced to terms of labor by dividing the amount of capital by wages. This inaccuracy in the use of words continues throughout the chapter.

In continuing his work, he supposes that $\frac{Q}{a+y} = nq$, or $Q = nq\,(a+y)$. If the capital $nq\,(a+y)$ is used in a productive process where n laborers are employed, then, assuming that the "product of labor" of each man equals p, and the wages of each equal $(a+y)$, Thünen holds

that the rate of interest is expressed by the formula $z = \frac{n[p-(a+y)]}{nq(a+y)} = \frac{p-(a+y)}{q(a+y)}$. By a manipulation of this formula he claims to prove that "z is the factor by means of which the relation of the efficiency of capital to that of human labor is expressed"; and hence he concludes, "We are herewith placed into position to reduce to terms of labor the co-operation of capital in the production of a commodity." Here he sums up in two sentences the result of his investigations in § 13. Notice, however, that above he spoke of reducing *capital* to terms of labor, while here he speaks of reducing the *co-operation of capital* to terms of labor,— two entirely different things, which we must keep distinctly separated. In a moment we shall find how Falck has fallen into error by confusing these two processes.

Now let us see what use Thünen makes of these results. To find the expression for natural wages, he resorts to the case of a number of laborers combining to lay out a marginal farm. One of the quantities that he uses to obtain his formula is nq, and the manner in which he obtains that quantity he states in these words: Suppose "the laying out of the farm required the year's labor of nq men. . . . Unquestionably, in order to provide a new farm, is needed not only labor, but also the use of capital: (but) according to § 13, we can reduce the co-operation of capital to terms of labor, and thus express the costs of laying out the farm entirely in terms of labor." *

What does Thünen mean by saying, according to § 13, the co-operation of capital may be reduced to terms of labor? Knapp understands him to signify that the co-operation of capital can be reduced to terms of labor by means of the rate of interest, and he states that such a reduction is impossible. This interpretation of Thünen's meaning I should urge as correct, and should base the claim (1) upon the general meaning of § 13, which is

* Part II., p. 152.

expressed in these sentences: "z is the factor by means of which the relation of the efficiency of capital to that of human labor is expressed," and "We are herewith placed into position to reduce to terms of labor the co-operation of capital in the production of a commodity"; (2) upon Thünen's use of words. In the paragraph under discussion he says, "According to § 13, we can reduce the co-operation of capital [*die Mitwirkung des Kapitals*] to terms of labor"; and in the latter of the two sentences just quoted he uses the identical expression, the co-operation of capital (*die Mitwirkung des Kapitals*).

Falck, however, takes exception to Knapp's interpretation, and states his objection by saying that in the first place, by dividing the value of capital by the value of a year's wages, the reduction of which Thünen speaks is possible; and in the second place, according to Thünen, the reduction is not effected by means of the rate of interest. Curiously enough, he informs the reader that, by referring to § 13 of Thünen's work, he may convince himself of the validity of his objection to Knapp's criticism. The truth of the matter is that Falck not only misinterprets both Thünen and Knapp, but he also misquotes Knapp. That he misinterprets those authors is evident from the fact that they speak of reducing the co-operation of capital to terms of labor, and Falck speaks of reducing capital to terms of labor. That he misquotes Knapp can be readily seen by comparing page 16 of Knapp with page 23 of Falck.*

If the interpretation that I have given of § 13 is correct,

*Knapp, p. 16: "Thünen bemerkt . . . das ein Grenzgut nicht nur durch Verwendung von Löhnen hergestellt werden kann; er glaubt aber . . . man brauche bloss 'die Mitwirkung des Capitals auf Arbeit zu reduciren,' so erreiche man das Gewünschte; diese Reduction, selbst wenn sie möglich wäre, geschieht nach Thünen nur durch den Zinsfuss, und da der noch zu finden ist, so bleibt also die Schwierigkeit ungelöst."

Falck, p. 23: "Schliesslich sagt Knapp, . . . 'Diese Reduktion (des Capitals auf Arbeit), selbst wenn sie möglich wäre,'" etc.

Knapp is speaking of the reduction of the co-operation of capital, and Falck quotes him as speaking of the reduction of capital.

it is easy to expose a fallacy in Thünen's reasoning that vitiates the formula $\sqrt{ap}$. Thünen's purpose in his whole work is to find mathematical expressions for the natural rate of interest and the natural rate of wages. The method by which he does this is first to find a formula expressing the interdependence of wages and interest in the isolated state. This formula $z = \frac{p-(a+y)}{q(a+y)}$ we shall call formula A. In this formula all the quantities are known except y and z. In order to find the values of y and z, he next attempts to find an independent expression for y, or what is the same thing, since a is known, an independent expression for $(a+y)$; and by substituting for $(a+y)$ in formula A obtain the value of z. The formula that enables him to find the independent expression for $(a+y)$ is $\frac{n[p-(a+y)]}{nq(a+y)}$. In this formula, which we shall call formula B, all the quantities are assumed as known excepting y. But how did Thünen obtain the quantity nq? He says: Suppose "the laying out of the farm required the year's labor of nq men. . . . Unquestionably, in order to provide a new farm, is needed not only labor, but also the use of capital; (but) according to § 13, we can reduce the co-operation of capital to terms of labor, and thus express the costs of laying out the farm entirely in terms of labor." When we refer to § 13 to see how the reduction is to be performed, we find that it is done by means of the rate of interest. The fallacy in the argument is evident. Thünen's whole procedure is a mere begging of the question. His problem is to find the values of y and z in formula A; and, to solve the problem, he undertakes to find an independent expression for $(a+y)$ by means of formula B, and by substituting for $(a+y)$ in formula A obtain the value of z. But, in order to get the quantity nq in formula B, he assumes that z is known. If, however, z is known, then, according to formula A, y is known. Thünen undertakes to find the value of the unknown quantities y and z; and, in attempting to solve the problem,

he uses the very quantities that he wants to find as known quantities.

While this error renders useless the formula $\sqrt{ap}$, yet it enables us to see why Thünen did not proceed to obtain the formula in the manner indicated by Komorzynski, and shows us the continuity in his work. In the part of his work based upon actual conditions (§ 13) he obtains the formula $z = \frac{n\,[p-(a+y)]}{nq\,(a+y)} = \frac{p-(a+y)}{q\,(a+y)}$; and in another part he states that it is to the advantage of the laborer to have the relation of wages and interest such that he can obtain the maximum income from his surplus when that surplus is placed at interest,— that is to say, it is to the advantage of the laborer to have yz obtain a maximum value. Now, Komorzynski holds that in these few facts we have the data with which to obtain the formula for natural wages, and that Thünen need not have carried his work farther; for, he says, if it is to the advantage of the laborer to have yz obtain a maximum value, then, since $z = \frac{p-(a+y)}{q(a+y)}$, his interests will be subserved when $\frac{y\,[p-(a+y)]}{q(a+y)}$ obtains a maximum value, which is the case when $(a+y) = \sqrt{ap}$. But Komorzynski assumes that the value of q in the above expression is known; and, in making this assumption, he has fallen into error. In § 13 the denominator of the expression for interest is obtained by dividing a definite amount of capital, Q, by the rate of wages. $\frac{Q}{a+y} = nq$ or $Q = nq\,(a+y)$. Hence nq is unknown as long as $(a+y)$ is unknown; and, when nq is known, $(a+y)$ is known, because their product is the definite quantity Q. Since, then, the value of q depends upon the value of $(a+y)$, it is an error to attempt to find the value of the unknown quantity $(a+y)$ in the above expression by assuming that q is known.

Thünen's method of obtaining the formula leads us to believe that he foresaw this difficulty. His sole object in trying to reduce the co-operation of capital in production to terms of labor was to enable him to proceed with his

work by considering nq a known quantity. When he attempted to find the expression for natural wages by assuming that a number of laborers combined to lay out a marginal farm, he did not begin by assuming that the value of the farm equalled Q, and by dividing that quantity by the rate of wages obtain nq; but he began by supposing that nq equalled the number of men required to lay out the farm. Knapp and Komorzynski * notice that he changed his method of obtaining nq, and, without seeing his purpose, charge him with inconsistency. By following his work, however, we find that he had a definite purpose in view; for, assuming that nq is known, he obtained an expression that is identical in form with the above expression from which Komorzynski obtains the formula $\sqrt{ap}$, but differs from it, in that nq, according to Thünen, is a known quantity.

Thünen's theory is valuable,† because it marks a decided reaction against the teachings of the classical economists, and yet at the same time avoids the extravagant doctrines of the socialists. His specific contribution to the theory of natural wages does not consist in his mathematical formulas, nor, indeed, in any positive conclusions that he obtains, but rather in his designation of the factors that must be considered in any scientific theory of natural or just wages. What these factors are can best be seen by contrasting Thünen's theory with that of the classical economists.

1. While the classical economists treated labor throughout as a mere commodity, Thünen regards the laborer as a *man*, and considers his wages as the means of satisfying

* Knapp, pp. 15, 25. Komorzynski, pp. 55, 56.

† It must be remembered that the first edition of *Der naturgemässe Arbeitslohn* appeared in 1850. For valuable suggestions concerning Thünen's contribution to Natural Wages, compare Schumacher's *Ueber J. H. v. Thünen's Gesetz vom naturgemässen Arbeitslohn*, pp. 18, 19; Mithoff (in Schönberg, *Handbuch*, Band 1), p. 640; Schmidt, pp. 2, 16.

his wants. The work that he undertakes is to find the wages that are agreeable to the nature (*naturgemäss*) and to the destiny of man.

2. While the classical economists regarded solely the operation of natural law, Thünen considers the equity in the case. He believes that natural wages exist in the isolated state when these two conditions are realized, (*a*) when the laborer receives the same income from his surplus when that surplus is placed at interest as the capital-producer receives from his surplus when that surplus is embodied in a marginal farm; (*b*) when the laborer receives the maximum income from his surplus.* Here Thünen makes a crude attempt to find an equitable basis for the division of the product of labor between the laborers and the owners of capital invested in concrete forms.

3. While the classical economists considered only the requirements of the laborer as limited by his surroundings, and disregarded the product of labor, Thünen holds that there can be no scientific theory of wages that does not make wages depend upon product. The fundamental idea in the formula $\sqrt{ap}$ is that wages must vary with the product.

A scientific theory of natural wages must regard the laborer as a man, consider the rights of the laborer and of the capitalist, and make the wages of the laborer depend upon his product.

* These two characteristics of natural wages are definitely stated, p. 204.

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* I have been unable to secure a copy of Brentano's dissertation. It is out of print.

Henry Ludwell Moore was born in
Harford County, Maryland, on November 21,
1869. He received his early education at
public and private schools in Baltimore.
He entered Randolph-Macon College, Vir-
ginia, in 1887, from which institution
he received his A. B. degree in 1892. Since
1892 he has pursued studies in econo-
mics, statistics, jurisprudence, and the-
ology at the Johns Hopkins University,
and the University of Vienna.